Two Styles of Modeling: Toward Linking Reaction Systems and Rough Sets

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Mathematics and the physical sciences made great strides for three centuries by ***constructing simplified models of complex phenomena****, deriving, properties from the models, and verifying those properties experimentally.*
This worked because the complexities ignored in the models were not the essential properties of the phenomena.
It does not work when the complexities are the essence.

Frederick P. Brooks [1]

Abstract. In the paper, we discuss two fundamentally different styles of modeling. In the first case, the models are designed by humans in the world of mathematics and next they are verified in the physical reality. In the second case, models are learned in interactions with the environment and they are continuously tuned using new acquired data and accumulated knowledge. After satisfactory interactions with the environment the model, which was not available to the designer a priori, rather is learned with time, provides a representation of the agent's environment. In this regard, the modelling needs to be based on the information acquired by agent-environment interaction. We discuss the first step in this direction by linking reaction systems with rough sets.

Key words: rough set, reaction system, interaction, interactive granular computing, modeling of complex states, modeling of transition relation on complex states

1 Introduction

Roughly speaking, a complex system can be understood as the one whose the elements are difficult to separate. Complex systems have been studied from a wide range of mathematical, scientific and engineering perspectives. A range of definitions have been developed to define and describe complex systems[7]. One may find examples of complex systems all around us [2, 3]: cells are composed of interacting molecules, brains are composed of interacting neurons, societies are composed of interacting individuals, ecosystems are composed of interacting species. We have already emphasized that in order to design complex systems, it is important to understand the concept of interaction [4, 5]. Without interactions, particular elements can become separated. However, when relevant interactions occur, the elements co-determine their future states. Thus, the future state of an element cannot be determined in isolation, as it co-depends on the states of other elements, precisely of those interacting with it.

Here, it is also worthwhile mentioning the remark, that the two main problems of Computer Science are related to the questions of what is the state and what is the transition relation, which Yuri Gurevich mentioned in one of his talk. This is especially important when we deal with states of complex systems. Let us also note that Murray Gell-Mann, the winner of Nobel prize in physics is discussing in the book [6] some difficulties in modeling of complex systems. He is also pointing out to the rule-based mathematics and agent based mathematics in searching for tools to deal with complex systems (see pages 320-321 in [6]).

Having the motto of this paper in mind, one may expect that the models of transition relations on states of complex systems, designed by humans, may not reflect the dynamics of complex systems. In particular, usually we have only partial, imprecise or imperfect information about states. Moreover, questions related to perception arise too. In particular, these are questions about the perception of states perceived by agents performing computations. The answers to such questions depend on the understanding of interactions of agents with the complex system embedded in the environment. Through interactions the agents can try to get satisfactory information for performing relevant actions toward achieving their goals. Here, one should resolve problems related to understanding interactions of physical objects to gain proper information about the environment in which the tasks are performed. Some progress in this direction has been made in Interactive Granular Computing (IGrC) (see, *e.g.*, [7–14]). IGrC is related to the problems mentioned in the following citations.

IGrC is strongly related to the opinion of Ch. L. Ortiz Jr. presented in [15]:

The Turing test, as originally conceived, focused on language and reasoning; problems of perception and action were conspicuously absent.

[7] `https://serc.carleton.edu/NAGTWorkshops/complexsystems/definitions.html`

Moreover, IGrC is also related to the opinion of Vapnik (see Epilog in [16]), where the need for considering the physical world as the basis for computations is expressed as follows.

> *further study of this [learning] phenomenon requires analysis that goes beyond pure mathematical models. As does any branch of natural science, learning theory has two sides:*
> - *The mathematical side that describes laws of generalization which are valid for all possible worlds and*
> - *The physical side that describes laws which are valid for our specific world, the world where we have to solve our applied tasks.*
>
> *[...] To be successful, learning machines must use structures on the set of functions that are appropriate for problems of our world. [...] Constructing the physical part of the theory and unifying it with the mathematical part should be considered as one of the main goals of statistical learning theory. [...] In spite of all results obtained, statistical learning theory is only in its infancy...*

According to Vapnik [16] there are many many aspects of this theory that have not yet been analyzed and that are important both for understanding the phenomenon of learning and for practical applications. Surely, one of the aspects should be to consider the necessity of linking the abstract world of mathematics with the physical world. This may be related to the grounding problem investigated in psychology [17–19, 14].

In the paper, we discuss two fundamentally different styles of modeling. In the first case, the models are designed by humans in the 'closed' world of mathematics and next they are verified in reality. In the second case, models are learned from gathered data and they are tuned using new data and accumulated knowledge. In this case, the model is not given a priori; it is learned with time through interactions with the physical reality. This is close to the idea of learning interaction rules presented by Valiant in [20].

In this paper, we restrict our considerations to a simplified version of IGrC assuming that objects are perceived by agents using some attributes. This means that the agents can obtain values of attributes (as the result of their interactions with the environment) on the basis of which they describe perceived objects. What is not covered in rough set approach [21–23] is the issue of methods of 'computing' the values of attributes on the perceived physical objects. IGrC in combination with rough set attempts to address the same. Here, one should use relevant searching methods for relevant parts of objects, learn methods to control the agent's attention to proper parts of the perceived objects, and identify which sensors and actuators as well as actions (plans) are to be chosen. We will present this more general approach in another paper.

Following the opinion by Brooks presented in the motto of the article, models of complex phenomena cannot be presented using the simplified exact models delivered by humans. However, one may ask how to develop methods for testing the quality of such models, or try to discover when applications of such models are relevant and how they can be adopted to changes in the environment.

Moreover, one may ask for aggregations of such models developed for different levels of hierarchical learning. This points the issue of learning so called interaction rules and strategies of their adaptation, which provides a way to tune the existing models with the perceived changes in the environment, in the context of IGrC. One may also recognize here some links with the zoom structures (see, *e.g.*, [24]). However, to proceed in this direction it is necessary to base the modeling on data acquired by interactions with the environment.

In this paper, we restrict our discussion to two approaches for modeling processes. The first one was developed for reaction systems, proposed for modeling of chemical and/or biological processes [25–27]. We call them exact models. The second one is based on the rough set approach [21–23]. In the case of the rough set approach the transition relation is learned from data gathered as the result of interactions of the 'agent' with the phenomena in the environment. The induced models are evolving in time using adaptation strategies. The rough set model of complex states and transition relation on such states is based on sets of induced rules discovered from data. This idea of obtaining interaction rules through interaction with physical reality can be presented bringing an analogy with obtaining *local logics* through *logic infomorphism* considered by Barwise and Seligman in the *information flow* approach [28] or with *views of knowledge* represented by data tables (see, *e.g.*, [29–32]). In these rules the rough set approximations of concepts involved in the exact models can be used. Changes of states are expressed through discovering of meta-rules describing changes of the sets of rules (local logics or different views of knowledge). Using this kind of modeling one may ask up to what degree the proposed exact model and in what kind of situations is correct. On the other hand this kind of modeling based on the rough set approach is showing that modeling of complex phenomena should be embedded into the more complex context not tractable by exact models. For example, one may derive rules with expressions dependent on approximations of properties rather that on their exact bivalent semantics. Certainly, there is also the necessity to develop methods for possible aggregations of the two kind of models studied in the paper.

The content of the paper is organized as follows. In Sect. 2 we present the basic concepts of reaction systems necessary for the definition of the transition relation. Rudiments of rough sets are included in Sect. 3. The rough set approach to modeling of complex states and transition relation of reaction systems is outlined in Sect. 4.

2 Reaction Systems

In this section, we recall the basic notions concerning reaction systems (see, *e.g.*, [25–27]). These definitions will next be crucial for our discussion on modeling. A biochemical reaction can take place if all of its reactants are present in a given state and none of its inhibitors is present; when a reaction takes place, it creates its products. This intuition leads to the following basic definitions.

Definition 1. *A reaction is a triplet $a = (R_a, I_a, P_a)$, where R_a, I_a, P_a are finite nonempty sets with $R_a \cap I_a = \emptyset$. If S is a set such that $R_a, I_a, P_a \subseteq S$, then a is a reaction in S.*

The sets R_a, I_a, P_a, and called the reactant set of a, the inhibitor set of a, and the product set of a, respectively.

Definition 2. *Let T be a finite set*

- *Let a be a reaction. Then a is enabled by T, denoted by $en_a(T)$, if $R_a \subseteq T$ and $I_a \cap T = \emptyset$. The result of a on T, denoted by $res_a(T)$, is defined by: $res_a(T) = P_a$ if $en_a(T)$, and $res_a(T) = \emptyset$, otherwise.*
- *Let A be a finite set of reactions. The result of A on T , denoted by $res_A(T)$, is defined by: $res_A(T) = \bigcup_{a \in A} res_a(T)$.*

The intuition behind T is that of a state of a biochemical system, *i.e.*, a set of biochemical entities present in the current biochemical environment. Thus a is enabled by T if T separates R_a from I_a, *i.e.*, $R_a \subseteq T$ and $I_a \cap T = \emptyset$. The result of a set of reactions A on T is cumulative, *i.e.*, it is the union of results of all individual reactions from A. In fact, $res_A(T) = \bigcup\{res_a(T) | a \in A \text{ and } en_a(T)\}$.

Definition 3. *A reaction system, abbreviated rs, is an ordered pair $\mathcal{A} = (S, A)$ such that S is a finite set, and $A \subseteq rac(S)$, where $rac(S)$ denotes the set of all reactions in S.*

Definition 4. *Let $\mathcal{A} = (S, A)$ be an rs and let $n \geq 0$ be an integer. An (n-step) interactive process in A is a pair $\pi = (\gamma, \delta)$ of finite sequences such that $\gamma = C_0, \ldots, C_n$ and $\delta = D_0, \ldots, D_n$, where $C_0, \ldots, C_n, D_0, \ldots, D_n \subseteq S$, $D_0 = \emptyset$, and $D_i = res_{\mathcal{A}}(D_{i-1} \cup C_{i-1})$ for all $i \in \{1, \ldots, n\}$.*

The sequence γ is the context sequence of π, denoted by $con(\pi)$, and the sequence δ is the result sequence of π, denoted by $res(\pi)$. Then the sequence $\tau = W_0, W_1, \ldots, W_n$ defined by $W_i = C_i \cup D_i$ for all $i \in \{0, \ldots, n\}$ is the state sequence of π, denoted by $st(\pi)$, with $W_0 = C_0$ called the initial state of π (and of τ) denoted $init(\pi)$ (and $init(\tau)$). If $C_i \subseteq D_i$ for all $i \in \{1, \ldots, n\}$, then we say that π (and τ) are context-independent. Note that for any n-step, context-independent interactive process, we can take $C_i = \emptyset$ for all $i \in \{1, \ldots, n\}$ without changing the state sequence.

The context sequence formalizes the intuition that, in general, an rs is not a closed system, and so its behavior is influenced by its "environment." Note that a context-independent state sequence depends only on the initial state W_0 and its length $(n + 1)$. Also, in a context-independent state sequence $\tau = W_0, \ldots, W_i, W_{i+1}, \ldots, W_n$, during the transition from W_i to W_{i+1} all entities from $W_i - res_{\mathcal{A}}(W_i)$ vanish. This reflects our assumption of no permanency: An entity from a current state vanishes unless it is produced/sustained by A. Clearly, if π is not context-independent, then an entity from a current state can be also sustained (thrown in) by the context (C_{i+1}). This feature is also a major difference with standard models of concurrent systems such as Petri nets.

3 Rudiments of Rough Sets

The rough set (RS) approach was proposed by Professor Zdzisław Pawlak in 1982 [21–23] as a tool for dealing with imperfect knowledge, in particular vague concepts. Over the years many methods based on rough set theory, alone or in combination with other approaches, have been developed.

The rough set approach seems to be of fundamental importance in artificial intelligence and cognitive sciences, especially in machine learning, data mining and knowledge discovery from databases, pattern recognition, decision support systems, expert systems, intelligent systems, multiagent systems, adaptive systems, autonomous systems, inductive reasoning, commonsense reasoning, adaptive judgement, conflict analysis etc.

Relationships of rough sets with many other approaches, such as fuzzy set theory, granular computing, evidence theory, formal concept analysis, (approximate) Boolean reasoning, multicriteria decision analysis, statistical methods, decision theory, matroids, have already been clarified by researchers [33]. Despite of the overlap with many other theories rough set theory may be considered as an independent discipline in its own right. There are reports on many hybrid methods obtained by combining rough sets with other approaches; A few such to name are soft computing (fuzzy sets, neural networks, genetic algorithms), statistics, natural computing, mereology, principal component analysis, singular value decomposition, and support vector machines.

The starting point of rough set theory is the indiscernibility relation, which is generated from the information about objects of interest (defined later in this section as signatures of objects). The aim of indiscernibility relation is to express the fact that due to lack of information (or knowledge) we are unable to discern some objects based on the available information (or knowledge). This entails that, in general, we are unable to deal with each particular object separately; rather we can only consider granules (clusters) of indiscernible objects as a fundamental basis for the theory.

From a practical point of view, it is better to define basic concepts of this theory in terms of data. Therefore we will start our considerations from a data set called an *information system*.

Suppose we are given a pair $\mathbb{A} = (U, \mathcal{A})$ of non-empty, finite sets U and $\mathcal{A}$, where U is the *universe* of *objects*, and $\mathcal{A}$ is a set consisting of *attributes*, *i.e.*, functions $a : U \longrightarrow V_a$, where V_a is the set of values of attribute a, called the *domain* of a. The pair $\mathbb{A} = (U, \mathcal{A})$ is called an *information system* (see, *e.g.*, [34]).

Any information system can be represented by a data table with rows labeled by objects and columns labeled by attributes. Any pair (x, a), where $x \in U$ and $a \in A$ defines the particular entry in the table indicated by the value $e(x, a)$ (or in other words $a(x)$).

Any subset B of $\mathcal{A}$ determines a binary relation $\mathcal{IND}_B$ on U, called an *indiscernibility relation*, defined by

$$x \ \mathcal{IND}_B \ y \text{ if and only if } e(x, a) = e(y, a) \text{ for every } a \in B, \tag{1}$$

where $e(x, a)$ denotes the value of attribute a for object x.

Obviously, $\mathcal{IND}_B$ is an equivalence relation. The family of all equivalence classes of $\mathcal{IND}_B$, *i.e.*, the partition determined by B, will be denoted by $U/\mathcal{IND}_B$, or simply U/B; an equivalence class of $\mathcal{IND}_B$, *i.e.*, the block of the partition U/B, containing x will be denoted as $[x]_B$ (or more precisely $[x]_{\mathcal{IND}_B}$). Thus in view of the data we are unable, in general, to observe individual object; rather we are forced to reason only about the accessible granules of objects with respect to available knowledge (see, *e.g.*, [35, 22, 36]).

If $(x, y) \in \mathcal{IND}_B$ we will say that x and y are *B-indiscernible*. Equivalence classes of the relation $\mathcal{IND}_B$ (or blocks of the partition U/B) are referred to as *B-elementary sets* or *B-elementary granules*. In the rough set approach the elementary sets are the basic building blocks (concepts) of our knowledge about reality. The unions of *B-elementary sets* are called *B-definable sets*.

For $B \subseteq A$ we denote by $Inf_B(x)$ the *B-signature* of $x \in U$, which is represented by the set $\{(a, e(x, a)) : a \in B\}$. Let $Inf_B(U) = \{Inf_B(x) : x \in U\}$. Then for any objects $x, y \in U$ the following equivalence holds: $x \mathcal{IND}_B y$ if and only if $Inf_B(x) = Inf_B(y)$.

This indiscernibility relation is further used to define basic concepts of rough set theory. The following two operations on sets $X \subseteq U$, given by,

$$\mathsf{LOW}_B(X) = \{x \in U : [x]_B \subseteq X\}, \tag{2}$$

$$\mathsf{UPP}_B(X) = \{x \in U : [x]_B \cap X \neq \emptyset\}, \tag{3}$$

assign to every subset X of the universe U respectively two sets $\mathsf{LOW}_B(X)$ and $\mathsf{UPP}_B(X)$, called the *B-lower* and the *B-upper approximation* of X. The set

$$\mathsf{BN}_B(X) = \mathsf{UPP}_B(X) - \mathsf{LOW}_B(X), \tag{4}$$

will be referred to as the *B-boundary region* of X.

If the boundary region of X is the empty set, *i.e.*, $\mathsf{BN}_B(X) = \emptyset$, then the set X is *crisp* (*exact*) with respect to B; in the opposite case, *i.e.*, if $\mathsf{BN}_B(X) \neq \emptyset$, the set X is referred to as *rough* (*inexact*) with respect to B. Thus any rough set, in contrast to a *crisp set*, has a non-empty boundary region.

We also write $\mathsf{LOW}_{\mathcal{B}}(X)$, $\mathsf{LOW}_{\mathcal{B}}(X)$, and $\mathsf{BN}_{\mathcal{B}}(X)$ instead of $\mathsf{LOW}_B(X)$, $\mathsf{UPP}_B(X)$, and $\mathsf{BN}_B(X)$, respectively , where $\mathcal{B} = (U, B)$.

Thus a set is *rough* (imprecise) if it has non-empty boundary region; otherwise the set is crisp (precise). Therefore with every rough set we associate two *crisp* sets, called *lower* and *upper approximation*. Intuitively, the lower approximation of a set consists of all elements that *surely* belong to the set, and the upper approximation of the set constitutes of all elements that *possibly* belong to the set. The *boundary region* of the set consists of all elements that cannot be classified uniquely as belonging to the set or as belonging to its complement, with respect to the available knowledge. This is exactly the idea of vagueness proposed by Gottlob Frege [37].

Information systems with distinguished attributes (decisions) are called *decision systems*. More formally, a decision system is a tuple $\mathbb{DT} = (U, \mathcal{A}, d)$, where $(U, \mathcal{A})$ is an information system and $d \in A$ is a distinguished attribute called *decision*. Attributes from $A \setminus \{d\}$ are called *conditional attributes* (or conditions).

4 Rough Set-based Modeling of Complex States and Transition Relations

Going back to reaction systems one may ask how the states are perceived. Due to their complex nature only a partial, imperfect information or/and knowledge about them may be perceived. Using the rough set approach one may assume that states are perceived using attributes. Let us discuss this in more detail.

For a given reaction $a = (R_a, I_a, P_a)$, first we consider the reactants from R_a. Assuming $R_a = \{r_1, \ldots, r_k\}$, one can consider for each reactant r_i a decision system $\mathbb{DT}(r_i) = (U, A_{r_i}, d_{r_i})$, where U is a sample of complex states (or rather pointers to them), A_{r_i} is a set of attributes used for perceiving objects from U, and d_{r_i} is a decision defined as the characteristic function of the property *reactant r_i is in state s*. The decision class $C(r_i) = \{s \in U | d_{r_i}(s) = 1\}$ is the set of states containing the reactant r_i. This class can be approximated using the information system $\mathcal{A}(r_i) = (U, A(r_i) \setminus \{d(r_i)\})$ what leads, using the rough set approach, to its lower approximation $\mathsf{LOW}_{\mathcal{A}(r_i)}(C(r_i))$, upper approximation $\mathsf{UPP}_{\mathcal{A}(r_i)}(C(r_i))$, and boundary region $\mathsf{BN}_{\mathcal{A}(r_i)}(C(r_i))$ [8]. These components of approximation of the decision class $C(r_i)$ describe the result of perception of the property *reactant r_i is in state s*. One can define the generalized decision $\delta_{C(r_i)}$ for the decision class $C(r_i)$ by $\delta_{C(r_i)}(s) = 1$ for $s \in \mathsf{LOW}_{\mathcal{A}(r_i)}(C(r_i))$, $\delta_{C(r_i)}(s) = 0$ for $s \in U \setminus \mathsf{UPP}_{\mathcal{A}(r_i)}(C(r_i))$, and $\delta_{C(r_i)}(s) = \{0, 1\}$ for $s \in \mathsf{BN}_{\mathcal{A}(r_i)}(C(r_i))$. This function can be extended on the indiscernibility classes of states or the signatures of states. These extensions will be also denoted by $\delta_{C(r_i)}$.

In the next step of modeling, for expressing how the set R_a is perceived we consider an aggregation of decision systems $\mathbb{DT}(r_i)$, which is also a decision system, defined as follows.

$$\mathbb{DT}(R_a) = (U(R_a), \{\chi_{\mathsf{LOW}_{\mathcal{A}(r_i)}(C(r_i))}, \chi_{\mathsf{UPP}_{\mathcal{A}(r_i)}(C(r_i))}, \chi_{\mathsf{BN}_{\mathcal{A}(r_i)}(C(r_i))}\}_{r_i \in R_a}, d(R_a)),$$

where

- $U(R_a)$ is the set of tuples $(Inf_{A(r_1)\setminus\{d(r_1)\}}(s), \ldots, Inf_{A(r_k)\setminus\{d(r_k)\}}(s))$ for $s \in U$,
- $\chi_{X_i}(u) = \mu_{X_i}(u_i)$ for $u = (u_1, \ldots, u_k) \in U(R_a)$, where μ_{X_i} is the characteristic function of the set of signatures of objects from X_i in $\mathcal{A}(r_i)$, and
- $d(R_a)$ denotes the (generalized) decision $\delta_{R_a}(u) = (\delta_{C(r_1)}(u_1), \ldots, \delta_{C(r_k)}(u_k))$ for $u = (u_1, \ldots, u_k) \in U(R_a)$; the value $\delta_{R_a}(u)$ of the generalized decision δ_{R_a} is representing the result of perception of the property *reactants from R_a are in the current state u*; note that for some reactants the perception may not give certain decision about their inclusion in the current state.

Let us note that different aggregations can be defined using operations of join with constraints [39, 40].

[8] Using the approach presented, *e.g.*, in [38] one can consider instead of approximation on samples of states the approximations extended on the universe of objects (which may contain new states not belonging to U).

Analogously one may consider process of modeling of perception of the property *the inhibitors from I_a are not in the current state* and the property that this condition is true for all considered reactions. Next, one may aggregate corresponding to these properties decision (information) systems for obtaining the system for representing the result of perception of all these properties.

In the same way it is possible to obtain decision systems representing properties such as *product p from P_a is in the current state*, *all products from P_a are in the current state*, *all products from P_a are in the current state for all considered reactions a*.

Next, one can consider aggregation of the already constructed decision systems to the decision system over pairs of states with the first component describing the current state and the second component describing the results of the reactions performed on this state. From such systems one can induce a set of rules (local logic, a view of knowledge represented in the system) describing properties of the second component depending on the properties of the first component. For example, the following rule (written in informal way) can be treated as a 'justification to a degree' for the discussed exact model for reaction systems:
if all reactants for each considered reaction are perceived
with certainty in the current state
and none of inhibitors for the considered reactions is perceived
in the current state
then all products of all considered reactions and only these products are
perceived with certainty after performing of all reactions.

In this rule the term *with certainty* means that the considered state belongs to the lower approximation of the relevant region. With this explanation one may easily write this rule formally using formulas expressing the relevant approximation regions. The truth of this rule in a given data table (information system) may be checked.

The presented above rules can be rewritten in a ore formal way using definable approximation regions:

$$en_a(x) \longrightarrow \bigwedge_{1 \leq l \leq n_a} \chi_{\mathsf{LOW}_{\mathcal{A}(p_l)}(C(p_l))}(y) = 1, \tag{5}$$

$$\bigwedge_{p \in P} \left[\chi_{\mathsf{LOW}_{\mathcal{A}(p)}(C(p))}(y) = 1 \longrightarrow \bigvee_{a \in A} (en_a(x) \ \& \ p \in P_a) \right], \tag{6}$$

where $\mathcal{A} = (S, A)$ = reaction system, x - the state before performing reactions from $\mathcal{A}$, y - the state after performing reactions from $\mathcal{A}$ in x, $R_a = \{r_1, \ldots, r_{k_a}\}$ - the reactant set of a, $I_a = \{i_1, \ldots, i_{s_a}\}$ - the inhibitor set of a, $P_a = \{p_1, \ldots, p_{n_a}\}$ - the product set of a, $P = \bigcup_{a \in A} P_a$, and

$$en_a(x) \equiv \left[\bigwedge_{1 \leq i \leq k_a} \chi_{\mathsf{LOW}_{\mathcal{A}(r_i)}(C(r_i))}(x) = 1 \ \& \ \bigwedge_{1 \leq j \leq s_a} \chi_{\mathsf{LOW}_{\mathcal{A}(i_j)}(C(i_j))}(x) = 0 \right].$$

Using the proposed modeling, one can expect to obtain a set of such rules describing not only exact dependencies between approximated regions. Moreover, one may use more advances methods of approximation such as Variable Precision Rough Set Model [41].

Finally, a model of transition relation can be represented by a set of rules. These models may change when the accumulated data are changing (by adding new states, attributes or methods of aggregation). Hence, one may also look for learning methods for prediction how such sets of rules are changing (on the basis of the accumulated data and knowledge). This problem of evolving models of transition relations with time when conditions in the environment are changing is one of the important issue to be studied. Let us also note that the discussed sets of rules (local logics, views of knowledge represented in data tables) may be used for inducing concurrent models consistent with such sets of rules (see, *e.g.*, [29–32]).

5 Conclusions

In the paper we discuss two approaches for modeling complex processes. The first one is based on exact models and the second one on the rough set approach. The proposed model based on the rough set approach seems to be also suitable for modeling situations related to different contexts in which reactions are performed as well as for learning dependencies between different levels of hierarchical modeling (such as modeling on the level of biochemical reactions in cells and the level of cells concerning behavioral patterns of cells). Further studies are needed to clarify the usefulness of the proposed approach in modeling complex phenomena occurring in real-life applications. Another problem to which the proposed approach seems to be very suitable concerns problems of control of reaction systems. Also links with other approaches like aggregation of information systems into networks of information systems with the information flow approach [28] and zoom structures [24] should be explored.

Acknowledgement

This work was partially supported by the Polish National Centre for R&D (NCBiR) under the grant DZP/RID-I-44/8/NCBR/2016.

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